A STATISTICAL ANALYSIS OF *GAMAKA*S IN CARNATIC MUSIC

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ABSTRACT

Carnatic Music, a form of classical music prevalent in South India, has a central concept called *rāga*s, defined as melodic scales and/or a set of characteristic melodic phrases. These definitions also account for the continuous pitch movement in *gamaka*s and micro-tonal adjustments to pitch values.

In this paper, we present several statistics of *gamaka*s to arrive at a model of Carnatic music. We draw upon the two-component model of Carnatic Music, which splits it into a slowly varying 'stage' and a detail, called 'dance'. Based on the statistics, we propose slightly altered definitions of two similar components called constant-pitch notes and transients. An automated implementation of these definitions is used in collecting statistics from 84 concert renditions.

We then suggest that the constant-pitch notes and transients can be considered as context and detail respectively of the *rāga*, but add that both are necessary for defining the *rāga*. This is verified by performing listening tests on only the constant-pitch notes and transients independently.

1. INTRODUCTION

Carnatic music, a classical form of music prevalent in South India, has evolved into a sophisticated art and a professional field. It uses *svara*s that roughly correspond to the twelve notes of the Western-music scale. *Svara*s are defined with respect to a base pitch called the *ādhāra ṣaḍja*, or the tonic. A subset of the 12 notes are played in specific orders and grouped together as phrases. These scales and phrases are used in defining *rāga*s.

Indian musicians use the regions 'between the notes' as a means of introducing sophistication over monophonic music. This can be seen in Figure 1 (reproduced with permission from [6]). The scales are thus notes connected by *gamaka*s that traverse frequencies between them; they can glide over even intermediate notes to reach notes further away.

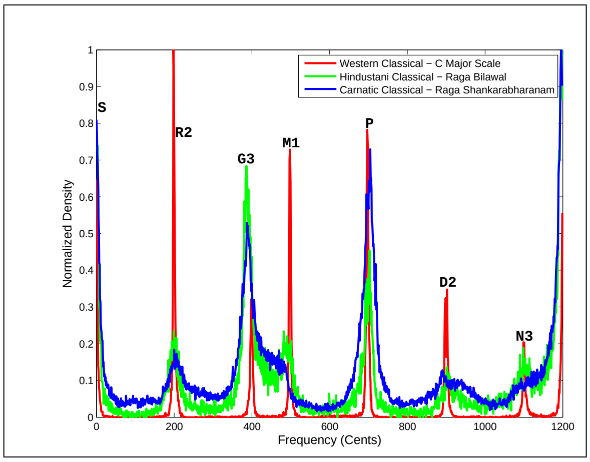

Figure 1: Histogram of notes of Carnatic, Hindustani and Western music renditions. Notice notes in Indian music span the regions between the notes of Western music.

The term *svara*, initially named the seven major notes in an octave – Sa, Ri (Carnatic)/Re (Hindustani), Ga, Ma, Pa, Da/Dha and Ni respectively corresponding to Do, Re, Mi, Fa, So, La and Ti in Western music. However, it has now expanded in meaning to identify even the transients between a sequence of notes.

The concepts of *gamaka* and *rāga* are central to Indian classical music. There is a perceived identity of *rāga*s that needs to be preserved. The characteristics of a *rāga* are determined in terms of its melodic phrases. Our observation is that a melodic phrase can be thought of as being made up of a sequence of constant-pitch notes (or CP-notes), and transients. Specific combinations of CP-notes and trajectories of the transients together give a *rāga* its unique identity. Further, we show that the constant-pitch notes serve as a context to interpret the transients in *gamaka*s. While the focus is primarily on Carnatic music, the methodology described could perhaps be extended to other genres that include significant continuous pitch movement, such as Hindustani music and possibly Jazz music.

The rest of the paper is organized as follows. Section 2 lists previous work and in Section 3, an extensive analysis of CP-notes and transients from concert recordings motivates our proposed model. Relevant previous work is described in terms of our model in Section 4.1. The results of a listening experiment are presented in Section 4.2 to verify the model and Section 5 concludes the paper.

2. PREVIOUS WORK

One of the earliest attempts of analyzing *gamaka*s in Carnatic music using signal processing techniques was [16]. After an initial analysis of a few *rāga*s, the author proposed the use of 'melodic atoms' to describe music. Again, as in the case of *gamaka*s, these are intuitive to the Carnatic musician, but there was no automated way of extracting them. At a similar time, *gamaka*s were analyzed for music information retrieval in [22]. This used the concept of 'extrema' of the pitch contour, which were called 'stationary points' in [8] for discovering *rāga* motifs.

Given the importance of *gamaka*s as borne out by Figure 1, it is not surprising that they were the subject of analysis in the CompMusic project [2]. An initial attempt was based on several types of *gamaka*s identified by musicologists [5]. A Carnatic musician, especially an instrumentalist, can intuitively relate to the descriptions of the movements, but looking for these pitch patterns in audio samples was not fruitful. Instead, *rāga* motifs (e.g. [8, 14]) were identified. *Rāga* motifs can be seen as signature pitch contour movements of a *rāga* and occur only in that *rāga* and no other. This exclusivity is an important aspect because, in Carnatic music, *rāga*s do share many common phrases and the unique phrases are expected to be small in number. A more recent, completely automated method of *rāga* identification based on motifs is presented in [13].

In [21], we find manual analysis of Carnatic music *gamaka*s, mainly for use in synthesis. However, it is based on one rendition of one *varṇam*. At a high level, it views Carnatic music as two components: the stage, which has a slowly varying pitch-contour, and the dance – a detail on top of the stage. We show in this paper that the variations in detail are quite short (typically under 400 ms) and propose different definitions of the two components so that automatic analysis is possible. In fact, [10] extends the stable part of the pitch contour to segments of varying pitch for Hindustani music retrieval; however, it is incomplete because significant information about (especially Carnatic) *rāga*s is in the segments of audio where the pitch varies (as we will show in Figure 5 later). A more recent result [11] uses quantized forms of these segments for Hindustani music, but does not characterize the variations.

In Section 4.2, the two-component theory, motifs and the context-detail aspects of Carnatic music will be revisited in the light of the model we propose.

3. ANALYSIS OF *GAMAKA*S IN CONCERT RECORDINGS

We analyzed 84 Carnatic music concert pieces in seven *rāga*s. The *rāga*-wise split of these pieces is given in Table 1. They are from the CompMusic database [1]. We chose seven *rāga*s from those considered as the major in Carnatic music [4] – *tōḍī*, *bhairavī*, *kharaharapriyā*, *kāmbhōji*, *śankarābharaṇam*, *varāḷī*, and *kalyāṇī*.

Rāga	Number of pieces	
	Male	Female
Tōḍī	7	5
Bhairavī	6	6
Kharaharapriyā	7	5
Kāmbhōji	8	4
Śankarābharaṇam	6	6
Varāḷī	7	5
Kalyāṇī	9	3

Table 1: *Rāga*-wise split of the 84 vocal-concert renditions.

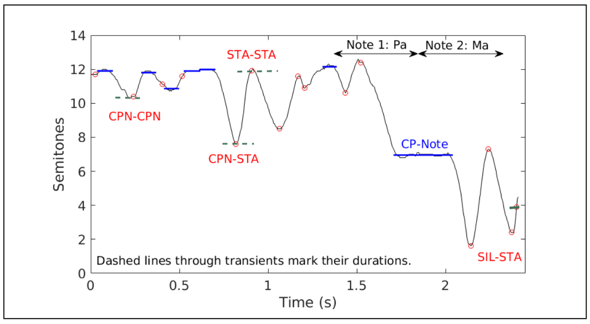

Figure 2: Examples of CP-notes (solid lines) and four types of stationary points (unfilled circles). The two notes marked exemplify EOb3.

3.1 Some Definitions

3.1.1 Constant-pitch Notes

Constant-pitch notes serve as a starting point for various musical forms including Carnatic music and we provide a working definition below, where the prefix EOb stands for 'Empirical Observation':

EOb1 A constant-pitch note is one whose pitch does not vary from its mean pitch by more than Δ semitones and lasts for at least $C_{\min}$ seconds.

where, nominally, $\Delta = 0.3$ and $C_{\min} = 80$ milliseconds (ms). The value of 80 ms is intuitive: it is the length of the shortest note played without *gamaka*s, in Carnatic music pieces that are extremely fast (*gamaka*s are not perceived at these speeds). This is possible on instruments for *rāga*s such as *mōhanam* (e.g. [3]) and *hamsadhwani*. Such renditions from a personal collection were analyzed. The duration of a CP-note at the fastest speed was found to be around 87 ms in one rendition and 80 ms in another. Only $\Delta = 0.3$ was consistent with this observation. For example, with $\Delta = 0.1$, the shortest CP-notes lasted more than 200 ms. Note that with other choices for Δ and $C_{\min}$, while the details of the statistics obtained may vary, the meaningful split of Carnatic music into CP-notes and transients is expected to hold.

3.1.2 Stationary Points

Stationary points were defined and used extensively in [7, 9, 22]. These are pitch positions where a continuous pitch curve changes direction. An example of these two features is shown in Figure 2. In the figure, a CP-note is easily recognized as a solid line drawn for its duration at its mean pitch value. The stationary points are marked by unfilled circles. (Note that statements 7 to 14 in Algorithm 2 remove redundant stationary points.) The same figure also shows a region of silence at the end of the time-axis. During silence, there is no prominent melodic pitch (i.e. neither of the voice nor of the accompanying violin), but a drone may be present. Four out of the six possible types of pitch curves – called transients – are marked in the figure. These types are defined by the neighbours of a stationary point and are in fact, the six combinations possible from the set $\{$CPNote, STAtionary point, SILence$\}$ (the letters in upper-case are the short-forms used later) :

1. CP-note on one side of the transient and, on the other, one of $\{$Another CP-note; A curve, which has a stationary point; Silence$\}$.
2. A curve with a stationary point on one side of the transient, and on the other, one of $\{$A curve with its own stationary point, Silence$\}$.

Our analysis counts the number of occurrences of each of the above and is presented next [1] .

3.2 Method

For each of the renditions in Table 1, the prominent pitch and tonic extracted for the work reported in [12] were re-used. They had been found using the algorithms described in [18], with a window length of 46 ms and a window shift of 2.9 ms [19]. Sometimes, the tracked pitch could be that of accompanying instruments and is prone to some octave-errors. However, unlike in transcription (e.g. [17]), these errors are inconsequential in identifying the type and duration of each transient (Section 3.1.2) or CP-note. The only impact was at most the splitting of a CP-note into more than one. Manual inspection of several pieces and their pitch tracks suggested that this occurred infrequently enough to be neglected.

Algorithm 1 was used to identify CP-notes and Algorithm 2, stationary points [2] . Each stationary point is also marked with one of the types described in Section 3.1.2.

Statements 4 to 9 of Algorithm 1 look for the longest possible CP-note from the current pitch value (at index i). However, in statement 6 , a direct application of the definition of CP-notes (Section 3.1.1) resulted in too many false alarms. For example, if the pitch rose by 0.6 semitones within 80 ms, but was part of a continuous pitch movement, it would still get detected as a CP-note. To eliminate these cases, a threshold of one semitone per second was used on the slope of the best-fit line. This is one-tenth the nominal slope of continuous pitch variation reported in [20].

The utility of defining CP-notes also emerges from Figures 3 and 4. Both *rāga*s show that the distribution of CP-notes is much sharper than all notes put together. (Ignore the stationary- and remaining-points for now.) For example, in Figure 3, notice when the notes are all plotted together (dashed line), they show a significant value (defined as occurring more often than 2% of the maximum occurrence) over G2 (3 semitones from Sa). However, the CP-notes show no such peak. This means there is no CP-note peak masked by the flat sections of the histogram. Further, there are no peaks found at 'incorrect' locations, say at 9.5 semitones. In fact, this behavior is sufficiently consistent to attach the name of the nearest semitone to the *svara*. That is, in a system of music with continuous pitch variation, *whether the CP-notes occur as part of svara*s *with gamaka*s *or without*, the CP-notes cluster around the *rāga*'s scale notes. This result is intuitive, but it is not obvious from, say, Figure 1.

In Figure 3, the automatically identified *svara*s are a subset of the written *ārohaṇa* and *avarohaṇa* of *śankarābharaṇam*. An interesting exception occurs in the *rāga tōḍī* (Figure 4) where the peak at R2 (2 semitones from Sa), is not part of its *ārohaṇa* and *avarohaṇa*.

Algorithm 1 Algorithm to find CP-notes from the pitch curve.

1: Track the pitch of each piece according to [18], which also identifies silence regions. These result in *pitch values*, $f[i], i \in \{0, \ldots, L-1\}$.
2: In the regions of music (i.e. not silence), with the tonic of the piece as f_0, find the pitch in semitones with respect to the tonic as $n[i] = 12\log_2(\frac{f[i]}{f_0})$
3: $i \leftarrow$ Index of the first pitch sample in the first non-silence region
4: **while** $i < L$ **do**
5:   flagCpNoteFound $\leftarrow$ FALSE
6:   **while** $n[j], j \in \{i + C_{\min}, \ldots k\}, j < L$ is a CP-note **and** the slope is below the threshold **do**
7:     $k \leftarrow k + 1$
8:     flagCpNoteFound $\leftarrow$ TRUE
9:   **end while**
10:   **if** flagCpNoteFound = TRUE **then**
11:     Mark the region from i to $k - 1$ as a CP-note
12:   **end if**
13:   $i \leftarrow k$
14:   **if** i is in a silence region **then**
15:     $i \leftarrow$ the index of the first pitch sample in the next non-silence region.
16:   **end if**
17: **end while**

3.3 Key Observations

One more step in the method remains to be described, but it needs to be motivated by a very significant observation by looking ahead at the results. Of all the $\approx 975,000$ transients that were found using Algorithm 2, only 1.25% were of the type SIL-STA-STA-STA-SIL, i.e. the central stationary point was flanked on both sides by two stationary points that ended in silence segments. Further, when one of the renditions was examined closely by ear (by a semi-professional musician), *all* such instances were false

[1] Silence on both sides of the transient was seen in under 0.005% of the cases and is excluded.

[2] A more sophisticated algorithm can be found in [7], but the inferences do not depend on precise locations of the stationary points.

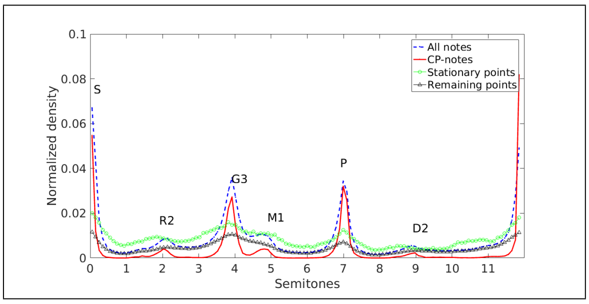

Figure 3: Histograms of all notes, CP-notes, stationary points and the remaining points in transients. *Svara* names are marked for significant peaks of the CP-notes' distribution. These peaks are much sharper than others. This is for the *rāga śankarābharaṇam*, whose nominal scale is S, R2, G3, M1, P, D2, N3, but N3 is not a significant peak.

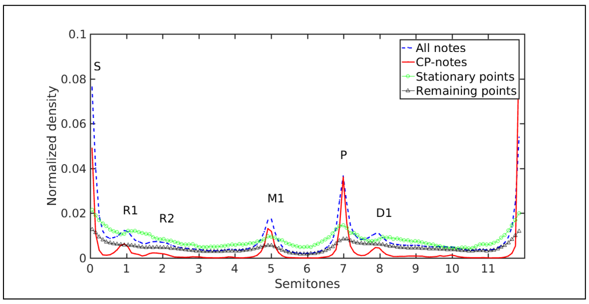

Figure 4: As in Figure 3, but this is for the *rāga tōḍī*. The nominal scale is S, R1, G2, M1, P, D1, N2, but G2 and N2 are not significant peaks, while R2 (not in the scale) is one.

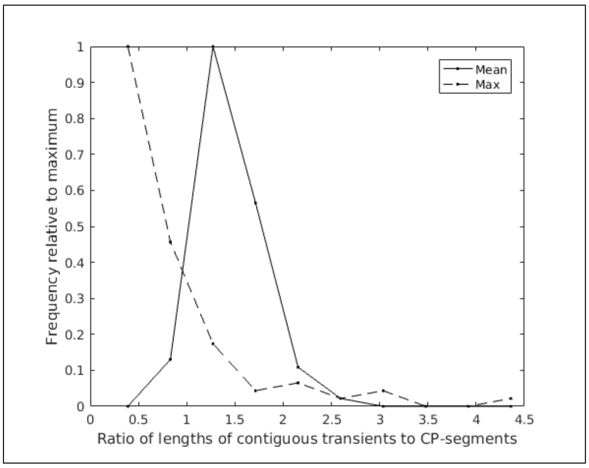

Figure 5: Ratios of maximum and mean durations of CCTs to that of CP-notes

alarms due to errors in marking silence-regions [3]. Thus, we may posit that:

EOb2 *Transients do not occur in isolation*. They occur along with a CP-note or 'chained' to other transients.

Further, from the intuitive experience of producing individual *svara*s *especially at very slow speeds*, we impose that individual *svara*s, when sung in isolation, need to have at least one CP-note segment. If this were not true, we should have had more instances of isolated transients. We use the term 'anchor note(s)' to denote the CP-note(s) associated with transient(s) in *svara*s.

Algorithm 2 Algorithm to find stationary points.

for Each segment of music (i.e. not silence) between CP-notes and silence (see Algorithm 1) **do**
  Smooth the pitch contour with a moving-average filter of length $L = \frac{C_{\min}}{4}$.
  Find the peaks and troughs of the pitch curve in the segment.
  for Each peak (trough) **do**
    Retain only the maximum peak (minimum trough) in a window of length L.
  end for
  for Each remaining peak OR trough, s, with pitch value n_s **do**
    if A nearest neigbhour of s is a CP-note with mean pitch value n_c **and** $|n_c - n_s| < \Delta$ **then**
      Discard stationary point s.
    else if The preceding neighbour is a stationary point s', with pitch value $n_{s'}$ **and** $|n_s - n_{s'}| < \Delta$ **then**
      Discard stationary point s'.
    end if
  end for
  Discard stationary points with pitch values between those of adjacent stationary points/CP-notes.
end for

If transients do not occur in isolation, it is instructive to see how long a 'contiguous chain of transients (CCTs)' can last without any CP-note appearing. A CCT is defined as a segment of music from the end of a silence segment/CP-note to the start of another. An example corresponds to the pitch curve in Figure 2 from $t \approx 0.65$ sec till $t \approx 1.3$ sec. The histograms of the ratios of the maximum and mean lengths of CCTs and CP-notes in the 84 pieces are shown in Figure 5. It reveals that *the maximum length of a contiguous chain of transients is comparable to the maximum duration of a CP-note* in that piece. Although there are cases where this ratio is > 2, in around 80% of the cases, it is < 1.5. Considering the mean durations: *Over* 50% *of the CCTs do not last longer than* 1.5 *times the mean CP-note duration*. Note that in this 'consistency check', the

[3] A similar percentage of CP-notes or transients flanked by SIL may thus be erroneous, but this does not affect the inferences made later.

definitions of the types of transients (Section 3.1.2) do not matter, so it is an independent corroboration of the utility of EOb2.

However, it does suggest the need to allow phrases to be concatenated by merging *svara*s. This type of chaining of phrases is familiar to musicians [15]. We further specify in detail:

EOb3 When two *svara*s are sung in sequence, their anchor notes of the same pitch can be merged.

Thus, while individual *svara*s need at least one anchor note each, S concatenated *svara*s can have $N(\leq S)$ CP-note segments in them. For example, the two notes – Pa and Ma – marked in Figure 2 share a common anchor note.

3.4 Statistical Analysis

Based on the key observations in Section 3.3, specific statistics relating to transients were collected. First, it is possible for merged anchor notes to reduce in duration and become stationary points in the limit. Thus, the starting point of a transient is counted as the nearest among {the end of a previous CP-note segment; another, earlier stationary point; or the end of a preceding silence segment}. Similarly, the end of the transient is nearest among the starting points of {a following CP-note segment; another, later stationary point; or the beginning of a succeeding silence segment}. These are marked in Figure 2. The distributions of the transient- and CP-note-durations across all *rāga*s are shown in Figure 6. The following observations are made.

1. The maximum duration of the transients is under 1 second, while the maximum duration of CP-notes is over 10 seconds. This result depends upon the definitions of starting points and ending points given above and Figure 5 vindicates the definitions.
2. The long transients are quite rare; more than 90% of them are shorter than 400 ms.
3. There is a variation in the ranges of the transient-durations across *rāga*s, but the mean values of the transient durations are remarkably similar ($\approx$ 100 ms) and is quite close to the 80 ms value set for the parameter $C_{\min}$. See Figures 7 and 8.
4. About 40% of the transients have a very small duration and need to be investigated further, but a major contributing factor are the 'attacks' of notes and where syllables are pronounced. During attacks, the spectrum changes and the pitch curve stops conforming to the definition of a CP-note (Section 3.1.1). Thus, *not all transients are perceived as gamakas.* Some may be perceived as *svara*-attacks.
5. The distribution of different types of stationary points defined in Section 3.1.2 is given in Table 2. This reaffirms the observations in Figures 3 and 4, where stationary points and other non-CP non-stationary points densities show wider peaks. The density for stationary points has been deliberately raised so that it is visible in the figures.

4. MODEL VERIFICATION

4.1 Relation to the Two-Component Model

The narrow peaks of the histograms of CP-notes (Figures 3 and 4) and the durations of transients being comparable to that of CP-notes (Figure 5) suggest that the CP-notes and transients identified by our method are meaningful. Thus, they can be compared with the model presented in [21], where the authors view Carnatic music as consisting of two components called the 'stage' and 'dance'. Our model was derived independently and it is quite significant that many aspects are similar, but there are important practical differences. First, 'stage' in [21] approximately maps to anchor notes, but we restrict anchor notes to be governed by EOb1. This enables automatic segmentation of the audio into CP-notes, transients and silence. Second, the stable and sustained focal pitches of the 'dance' may get classified as a CP-note in our model if they satisfy EOb1. The transient focal pitches would get classified as our transients. In general, focal pitches that do not satisfy EOb1 will be counted as transients. However, similar to [21], it is possible to view the CP-notes (approximately stage) as context of the music and the transients (at least the dance component) as its detail. Similarly, the oscillatory continuity condition of [20] can be seen as a special case of EOb3.

Note that while [21] was based on a largely manual analysis of one rendition of one *varṇam* in one *rāga* (in aid of synthesis), we have found approximate stage-like and dance-like components of 84 pieces in seven major *rāga*s using an automated method (Sections 3.2 and 3.3).

4.2 Listening Tests

We report the results of experiments designed to evaluate the dependence of transients and CP-notes [4]. Approximately 30-second snippets of violin *ālāpana*s (non-*tāḷa*-bound extempore improvisations in a *rāga*) were chosen for the test. Songs were excluded because listeners could have recognized the song's tune rather than the *rāga*. Each snippet was then split into a CP-notes-only and transients-only parts according to Algorithms 1 and 2. *Rāga*s *tōḍī*, *bhairavī*, *śankarābharaṇam*, *varāḷī*, and *kalyāṇī* were chosen and in each, two snippets were picked manually. The listener was asked to identify the *rāga*s of these 30 pieces, while a superset of these *rāga*s was given as possible choices. Listeners could also choose 'Cannot make out' if they actually could not or if they felt the *rāga* played was not in the list. The order of pieces was randomized separately for the transients-only (this set of 10 was played first), CP-notes only (played next) and the unedited snippets (played last). Fifty participants took the test.

The overall results point to the CP-notes' and transients' dependencies on each other: 81.4% of the clips were identified correctly, but only 58.4% of the transients-only snippets, and 68.6% of the CP-notes only. *Rāga*-wise results are presented in Table 3, which are restricted to the expert participants – those who could identify the *rāga*s of

[4] http://www.iitm.ac.in/donlab/wermusic/index.html?owner=venkat&testid=test1&wer=30

Type	CPN ∘ CPN	CPN∘STA	CPN∘SIL	STA∘STA	STA∘SIL
Percentage	14.5	17.5	5.3	49.0	13.7

Table 2: Relative occurrence of stationary-point types.

Rāga	Percentage of participants	
	Transients only	CP-notes only
Tōḍī	95.0	81.3
Bhairavī	43.2	81.1
Śankarābharaṇam	21.6	85.1
Varāḷī	86.5	89.2
Kalyāṇī	89.7	68.0

Table 3: Accuracy of identification by experts

all unedited snippets of that *rāga*. Only one of them could identify all instances of snippets with transients only and CP-notes only. Among the *rāga*s, *tōḍī* and *varāḷī* fared best. *Bhairavī* was surprisingly not easily identifiable with only transients, even though it is considered a *gamaka*-heavy *rāga*. *Kalyāṇī* was easier to recognize by transients than only CP-notes, but *śankarābharaṇam* could be identified from transients-only by only about 20% of the experts. Overall, the results suggest that CP-notes and transients need each other in Carnatic music.

Some qualitative results are also significant. A few participants expressed surprise when presented with only CP-notes or transients. Several said that the edited pieces were quite unpleasant to the ear (edited vocal pieces sounded worse). The edited pieces were noticeably fragmented, but the artefacts were identical in both types of clips. Yet. interestingly, the transients-only clips – a crucial component of *gamaka*s – were perceived as more unpleasant. This clearly suggests the importance of context for transients in Carnatic music, which is provided, at least partially, by the CP-notes.

5. CONCLUSION

The qualitative stage and dance model model for Carnatic music is corroborated with an equivalent CP-note, transient model. An analysis of CP-notes and transients shows that the CP-notes can last much longer than transients in duration. We proposed that *svara*s can be viewed as CP-notes providing the context for any transients. When combined with the chaining rule, these observations can explain *rāga* motifs.

In a listening test, while 28 experts correctly identified all original, unedited audio clips in five *rāga*s, only one correctly identified all CP-notes-only and transients-only clips. Several listeners reported that the latter pieces were quite unpleasant on the ear.

Thus, the CP-notes and transients model is potentially a good model of Carnatic music, with our analyses suggesting that there is significant contextual *rāga* information in CP-notes; they are, in fact, crucial for a pleasant listening experience of the transients in the profusion of *gamaka*s in Carnatic music.

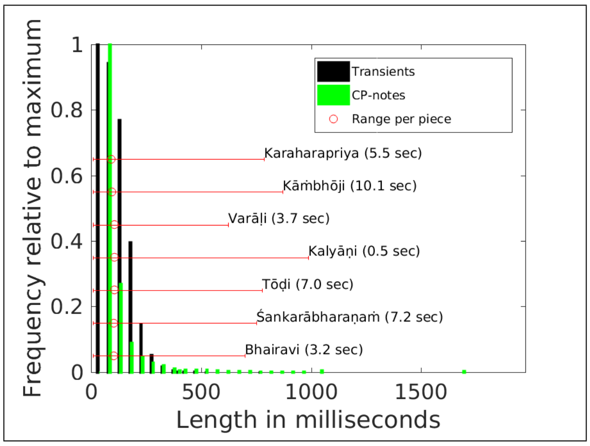

Figure 6: Distribution of the durations of transients and CP-notes across *rāga*s described in Table 1. Each horizontal line shows the *rāga*-specific range of transient-durations, and the circles, their mean durations. Numbers in brackets are the durations of the longest CP-note.

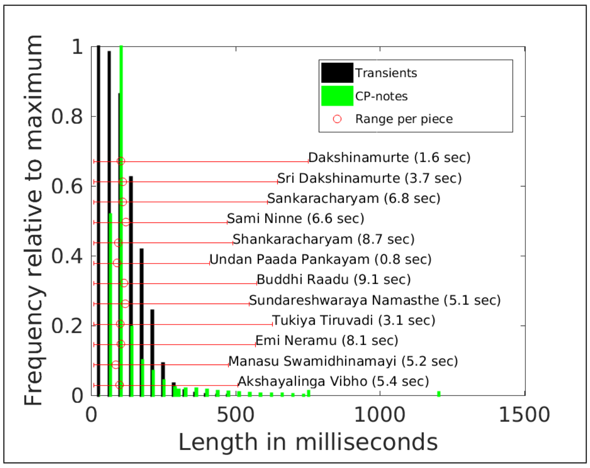

Figure 7: Distribution as in Figure 6 restricted to the *rāga* *śankarābharaṇam*. The range and means of transient-durations correspond to each piece in the *rāga*.

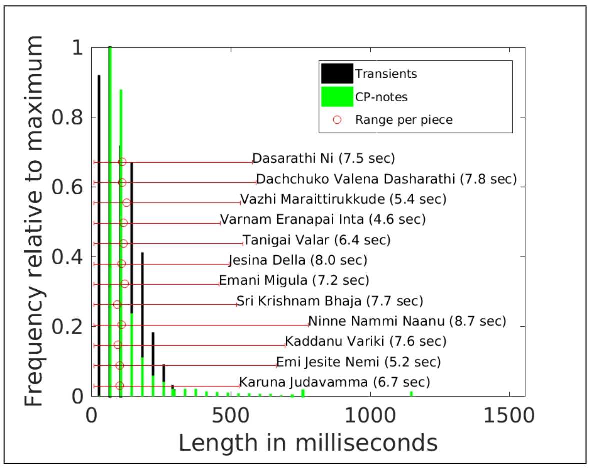

Figure 8: Distribution as in Figure 7 restricted to the *gamaka*-laden *rāga* *tōḍī*. Its transient-duration means are similar to those of *śankarābharaṇam*.

6. ACKNOWLEDGEMENTS

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